

Interactions Suppress Quasiparticle Tunneling at Hall Bar Constrictions

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Tunneling of fractionally charged quasiparticles across a two-dimensional electron system on a fractional quantum Hall plateau is expected to be strongly enhanced at low temperatures. This theoretical prediction is at odds with recent experimental studies of samples with weakly-pinched quantum-point-contact constrictions, in which the opposite behavior is observed. We argue here that this unexpected finding is a consequence of electron-electron interactions near the point contact.

Introduction: One-dimensional fermion systems have attracted enduring interest because they are converted from Fermi liquids to Luttinger liquids (LL)¹ by interactions. Convincing experimental evidence of LL behavior has been discovered in a number of quasi-one-dimensional systems, including carbon nanotubes², semiconductor quantum wires³, and the edges^{4,5} of incompressible two-dimensional electron systems on quantum Hall plateaus. From a theoretical point of view, quantum Hall edge systems are especially interesting⁶ because they do not reduce to Fermi-liquids even when interactions between charge fluctuations along the edge are neglected. Although edge systems can be complex,^{7,8} those that surround the simplest fractional quantum Hall state at $\nu = 1/3$ appear^{4,9} to be reasonably well described by the simplest consistent model which has a single chiral edge channel. A key prediction¹⁰ of theory is that tunneling of fractionally charged quasiparticles across a constriction in a two-dimensional electron gas, like the one illustrated schematically in Fig. 1, is universally enhanced at small bias voltages and low temperatures. This Letter is motivated by recent experiments^{11,12} in which the opposite behavior is consistently observed. The discrepancy exists even though theory appears to describes other non-trivial edge-related properties correctly, including the decrease in current noise due to the fractional quasiparticle charge¹³ and the suppressed tunneling density-of-states.⁹ In this Letter we argue that these surprising observations are a consequence of interactions near the constriction.

In order to achieve tunneling of fractionally charged quasiparticles it is necessary to bring opposite edges of a Hall bar into proximity by placing a split gate over the top. A typical sample has an overall length and width $\sim 100\mu\text{m}$, and a gate width which is ~ 100 times smaller. Simply put, our idea is that the edge states on the left and right sides of the split gate should be regarded as the counter-propagating chiral channels of a one-dimensional fermion system defined on the constriction, rather than as the left and right portions of the top or bottom chiral channels of the overall Hall bar. As we show, interactions across the split-gate suppress quasiparticle tunneling across the point-contact formed by its opening. We argue that these interactions can be sufficiently strong to

make weak quasiparticle tunneling an irrelevant perturbation and produce the low-bias tunneling *suppression* seen in many experiments. We elaborate on this idea by studying a simple toy model that captures essential features of the experimental geometry.

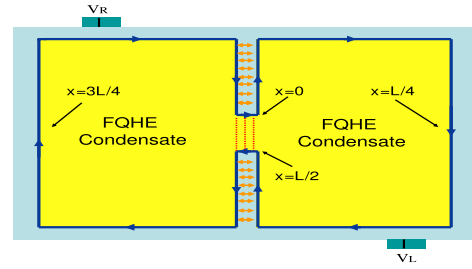


FIG. 1. Schematic illustration of a Hall bar with a split-gate constriction. Samples similar to this have been used to study tunneling of fractionally charged quasiparticles across an incompressible two-dimensional electron gas region with filling factor $\nu = 1/3$. For weakly pinched gates the $\nu = 1/3$ plateau extends through the point-contact. Our model measures position clockwise along the edge starting from the top of the split gate at $x = 0$, so that the bottom of the split gate is at $x = L/2$ where L is the total edge perimeter and positions are understood to be defined modulo L . Quasiparticle tunneling from top to bottom across the point contact is equivalent to electron backscattering by the constriction. The quasiparticle tunneling current I_{Tunn}^{qp} leads to a voltage drop $V = hI_{Tunn}^{qp}/e^2\nu$ across the constriction and to an identical deviation from perfect quantization for Hall voltages measured on either left or right hand sides of the Hall bar.

The model: We consider a Hall bar with a constriction whose edge encloses a singly connected area in which an incompressible quantum Hall state with filling factor ν has been established. (See Fig. [1].) Low energy physics on a quantum Hall plateau⁶ may be described in terms of an edge density collective coordinate $\rho(x)$. The effective Hamiltonian

$$H = \frac{1}{2} \int_0^L dx \int_0^L dx' \rho(x) U(x, x') \rho(x') \quad , \quad (1)$$

where $U(x, x') = \delta^2 E[\rho(x)] / \delta \rho(x) \delta \rho(x')|_0$. $U(x, x')$ can

be expressed as a sum of microscopic exchange and confining potential interactions that are expected¹⁴ to cancel approximately, and the Coulomb energy given approximately by $U(x, x') \approx e^2/[\epsilon|\vec{r}(x) - \vec{r}(x')|]$ where $\vec{r}(x)$ is a two-dimensional coordinate at the edge. Following arguments first articulated by Wen,⁶ it follows from the fractional quantum Hall effect that the single-chiral-channel model is quantized by imposing the commutation relations

$$[\rho(x), \rho(x')] = -(i\nu/2\pi)\partial_x\delta(x - x') \quad (2)$$

The quasiparticle tunneling operator is expressed below in terms of the chiral boson field $\phi(x)$, related to the density (in our convention) by $\rho(x) = \nu\partial_x\phi(x)/2\pi$.

With these approximations the quadratic edge action is completely specified by the sample geometry allowing any density-fluctuation correlation function to be evaluated numerically. Our argument is most simply explained, however, by considering a simple toy model for which the relevant correlation functions can be evaluated analytically. We consider the Hamiltonian $H = H_0 + H_1 + H_2$ where

$$H_0 = \pi v_F \int_0^L dx \rho(x) \rho(x), \quad H_1 = g_1 \pi v_F \int_0^L dx \rho(x) \rho(L - x), \\ H_2 = g_2 \pi v_F \int_0^L dx \rho(x) \rho(L/2 - x). \quad (3)$$

In Eq. (3) g_1 and g_2 are dimensionless interaction parameters, H_0 accounts for interactions between nearby points along the edge, H_1 for interactions across the split-gate, and H_2 for interactions between the top and the bottom of the Hall bar. For a Hall bar with a constriction we expect that g_1 is close to but smaller than 1 and that g_2 is close to 0. The parameter which characterizes the strength of local interactions, v_F , is the edge wave velocity for $\nu = 1$ and $g_1 = g_2 = 0$. The action for this model,

$$S = \frac{1}{L} \sum_{i>0} \int_{\tau} \left[\phi^*(q_i, \tau) \frac{i\nu q_i}{2\pi} \partial_{\tau} \phi(q_i, \tau) + \frac{\nu^2 q_i^2 v_F}{2\pi} \left[|\phi(q_i, \tau)|^2 - (g_1 + (-)^i g_2) \left(\phi^*(q_i, \tau) \phi^*(q_i, \tau) + \text{c.c.} \right) \right] \right], \quad (4)$$

has normal modes with eigenenergies and operator formalism creation operators given by $E_{n\pm} = \hbar v_F q_n \nu [1 - g_{\pm}^2]^{1/2}$ and $A_n = \cosh(\theta_{\pm}) a_n + \sinh(\theta_{\pm}) a_n^{\dagger}$. Here a_n and a_n^{\dagger} are edge wave creation and annihilation operators for $g_i = 0$ which are proportional to the $q = \pm 2\pi n/L$ Fourier components of the edge density, the plus signs apply for even n , the minus signs for odd n , $\tanh(2\theta_{\pm}) = g_{\pm}$ and $g_{\pm} = g_1 \pm g_2$.

Quasiparticle Tunneling I-V Characteristics: The operator which creates a quasiparticle of charge ν at the edge of the system is⁶ $\hat{\psi}_{qp}(x) = e^{i\nu\phi(x)}/(2\pi)^{1/2}$. The operator for quasiparticle tunneling from top to bottom across the constriction is therefore

$$T_{qp}(0, \frac{L}{2}; t) = \frac{1}{2\pi} \exp\{i\nu\phi(\frac{L}{2}, t)\} \exp\{-i\nu\phi(0, t)\} = \\ \exp\left[-i2e^{\theta} - \sum_{n>0}^{\text{odd}} \sqrt{\frac{\nu}{n}} \left[A_n e^{-iE_n t/\hbar} - A_n^{\dagger} e^{iE_n t/\hbar} \right]\right]. \quad (5)$$

Note that this operator depends only on the interaction combination $g_1 - g_2$; repulsive interactions across the constriction play the same role as attractive interactions across the Hall Bar.

The quasiparticle tunneling current can be evaluated by treating the tunneling term in the Hamiltonian, $H_{\text{Tunn}} = (\Gamma T_{qp} + \Gamma^* T_{qp}^{\dagger})$, perturbatively. At leading order this gives the usual Fermi Golden rule result:

$$I_{\text{Tunn}}^{\text{qp}} = \left[1 - \exp\left(-\frac{eV}{k_B T}\right) \right] \frac{e^*}{\hbar} \int_{-\infty}^{+\infty} dt e^{ieVt} G_{\text{Tunn}}^{\text{qp}}(t), \quad (6)$$

where the quasiparticle tunneling correlation function is

$$G_{\text{Tunn}}^{\text{qp}}(t) = \langle T_{qp}^{\dagger}(t) T_{qp}(0) \rangle = \frac{|\Gamma|^2}{(2\pi)^2} \exp\left[-4\nu e^{2\theta} - \sum_{n>0}^{\text{odd}} \frac{1}{n} \left[(n_B(E_n) + 1) \left(1 - e^{-\frac{iE_n t}{\hbar}} \right) + n_B(E_n) \left(1 - e^{\frac{iE_n t}{\hbar}} \right) \right] \right], \quad (7)$$

and $n_B(E)$ is the Bose distribution function. For $k_B T > \hbar v_F/L$ the sum can be replaced by an integral and we find (up to a constant factor) that

$$G_{\text{Tunn}}^{\text{qp}}(t) = \frac{|\Gamma|^2}{(2\pi)^2} \left(i \frac{v_- \beta}{\pi} \sinh \frac{\pi t}{\beta} \right)^{-2\nu e^{2\theta}}. \quad (8)$$

(For finite system sizes the quasiparticle tunneling correlation function takes the form

$$G_{\text{Tunn}}^{\text{qp}}(t) = \frac{|\Gamma|^2}{(2\pi)^2} F^{-2\nu e^{2\theta}}(z_-) \tilde{F}^{2\nu e^{2\theta}}(z_-), \quad (9)$$

where $z_- = v_- t$ and F and \tilde{F} are Euler elliptic ϑ -function ratios: $F(z) = \vartheta_1(\pi z/L | i\nu\beta/L) / \vartheta_1(-i\pi\alpha/L | i\nu\beta/L)$, and $\tilde{F}(z) = \vartheta_2(\pi z/L | i\nu\beta/L) / \vartheta_2(-i\pi\alpha/L | i\nu\beta/L)$ and α is an infinitesimal used to regularize the n -summation.)

Fourier transforming the large L correlation function yields the final expression for the tunneling current:

$$I_{\text{Tunn}}^{\text{qp}} = \frac{2e^* |\Gamma|^2 \sin(\pi d)}{\pi v_- \hbar} \left(\frac{v_- \beta}{2\pi} \right)^{1-2d} \text{Im} \left\{ B\left(d - i \frac{eV\beta}{2\pi}, 1 - 2d\right) \right\}, \quad (10)$$

where B is the Euler beta function. The key quantity in this expression is $d = \nu e^{2\theta}$ which is the scaling dimension of the tunneling operator.

The significance of d in the quasiparticle tunneling current expression is more apparent in the simpler expressions that apply in low and high temperature limits. For $eV\beta/2\pi \ll 1$

$$I_{\text{Tunn}}^{\text{qp}} = \frac{e^* |\Gamma|^2}{v_- \hbar} \left(\frac{v_- \beta}{2\pi} \right)^{1-2d} \frac{\Gamma^2(d)}{\Gamma(2d)} \cdot \frac{eV\beta}{2\pi} \left[1 + \left(\frac{\pi^2}{6} - \zeta(2, d) \right) \left(\frac{eV\beta}{2\pi} \right)^2 + \dots \right], \quad (11)$$

where $\zeta(2, d)$ is the Riemann zeta function. Note that $\zeta(2, d)$ is a monotonically decreasing function of d and equals $\pi^2/6$ for $d = 1$. It follows that for $d < 1$ the conductance dI^{qp}/dV diverges with T like T^{2d-2} in the Ohmic region, and decreases like $T^{2d-4}V^2$ at higher voltages. In the opposite case when $d > 1$ the low bias conductance goes to zero like T^{2d-2} in the Ohmic region, and increases like $T^{2d-4}V^2$ at higher voltages. In the high bias limit, $eV\beta/2\pi \gg 1$,

$$I_{\text{Tunn}}^{\text{qp}} = \frac{e^*|\Gamma|^2}{h\nu_-^{2d}} \frac{(eV)^{2d-1}}{\Gamma(2d)} \quad , \quad (12)$$

so that the conductance varies as V^{2d-2} . For non-interacting electrons on the integer Hall edge $\nu = 1$, $g_1 = g_2 = 0$, $d = 1$, and the tunneling conductance is independent of both bias voltage and temperature. The chiral LL prediction¹⁰ that fractional quasiparticle tunneling increases at low temperatures follows from the factor of ν in the expression for d which makes $d < 1$ and backscattering relevant. The key observation here is that interactions can alter this prediction if $e^{2\theta_-}$ is sufficiently large.

Interaction Parameter and Scaling Dimension Estimates: To determine whether or not interactions can alter the relevance of quasiparticle tunneling we need to estimate $\exp\{2\theta_-\} = [(1+g_-)/(1-g_-)]^{1/2}$. Because the microscopic Coulomb interactions are long ranged the effective interaction strengths depends weakly (logarithmically) on wavevector. The local interaction parameters in our model, v_F , is the Coulomb interaction Fourier transform cut-off at short distances by w_{edge} , the width of the region near the edge over which the charge density falls from its bulk value to zero. It follows that $v_F(k) \sim -e^2 \ln(kw_{\text{edge}})/\epsilon\pi\hbar$. Non-local interactions across the constriction and across the Hall bar are given by similar expressions, except that the short distance cut-offs are respectively the width of the constriction and the width of the Hall bar: $v_F(k)g_1 \sim -e^2 \ln(kw_{\text{sg}})/\epsilon\pi\hbar$ where $w_{\text{sg}} \sim 0.3\mu\text{m}$ is the width of the split gate, and $v_F(k)g_2 \sim 0$ except for the case of long thin Hall bars. For quantitative estimates it is important to realize that ϵ should be taken as the mean¹⁵ of the dielectric constant of the (GaAs) host semiconductor and vacuum. The wavevector $k = 2\pi n/L$ that is relevant to the experimental behavior is determined by the condition $E_{n-} \sim k_B T$, which defines a thermal length L_T with a value $\sim 100\mu\text{m}$ for temperatures ~ 0.1 Kelvin, in the middle of the typical measuring range. Since this length is not substantially larger than the length of the split gate, quasiparticle tunneling should be sensitive only to interactions along the gate and our idealized model is realistic. It follows that $g_1 \sim \ln(L_T/w_{\text{sg}})/[\ln(L_T/w_{\text{sg}}) + \ln(L_T/w_{\text{edge}})] \sim 0.84$, and that $\exp(2\theta_-) \sim 3.4$, larger than the value required to transform quasiparticle tunneling

The closed constriction limit: Similar calculations can be performed for electron tunneling between the disconnected regions on the left and right hand sides of the

Hall bar that enclose FQH liquids with filling fractions ν_L and ν_R respectively. We find that the *electron* tunneling-tunneling correlation function is

$$G_{\text{Tunn}}^{\text{el}}(0, \frac{L}{2}; t-t') = \frac{|\Gamma|^2}{(2\pi)^2} \cdot \left(i \frac{\nu_L \beta}{\pi} \sinh \frac{\pi(t-t')}{\beta} \right)^{-\frac{1}{\nu_L} (\cosh 2\theta - \sqrt{\frac{\nu_L}{\nu_R}} \sinh 2\theta)} \cdot \left(i \frac{\nu_R \beta}{\pi} \sinh \frac{\pi(t-t')}{\beta} \right)^{-\frac{1}{\nu_R} (\cosh 2\theta - \sqrt{\frac{\nu_R}{\nu_L}} \sinh 2\theta)} \quad . \quad (13)$$

When $\nu_1 = \nu_2 \equiv \nu$ and for large system sizes we find that the scaling dimension is $d^{\text{el}} = \exp\{-2\theta_-\}/\nu$. Since $d d^{\text{el}} = 1$, the top-bottom quasiparticle tunneling process for an open constriction is relevant whenever left-right *electron* tunneling through a closed constriction is irrelevant and vice-versa.

Discussion: The model we study here has a number of interesting symmetries that are captured by the general quasiparticle-quasiparticle correlation function

$$\langle R_{\text{qp}}^\dagger(x, t) R_{\text{qp}}(x', t') \rangle = \quad (14)$$

$$\left[F^{-\frac{\nu}{2} \cosh^2 \theta_+} (z_+ - z'_+) F^{-\frac{\nu}{2} \cosh^2 \theta_-} (z_- - z'_-) \right] \cdot \left[\tilde{F}^{-\frac{\nu}{2} \sinh^2 \theta_+} (\bar{z}_+ - \bar{z}'_+) \tilde{F}^{-\frac{\nu}{2} \sinh^2 \theta_-} (\bar{z}_- - \bar{z}'_-) \right] \cdot \left[\frac{\tilde{F}^{-\frac{\nu}{2} \cosh^2 \theta_+} (z_+ - z'_+) \tilde{F}^{-\frac{\nu}{2} \sinh^2 \theta_+} (\bar{z}_+ - \bar{z}'_+)}{\tilde{F}^{-\frac{\nu}{2} \cosh^2 \theta_-} (z_- - z'_-) \tilde{F}^{-\frac{\nu}{2} \sinh^2 \theta_-} (\bar{z}_- - \bar{z}'_-)} \right] \cdot \left[\frac{F^{-\frac{\nu}{2} \sinh \theta_+ \cosh \theta_+} (z_+ + \bar{z}'_+) F^{-\frac{\nu}{2} \sinh \theta_- \cosh \theta_-} (z_- + \bar{z}'_-)}{F^{-\frac{\nu}{2} \sinh \theta_+ \cosh \theta_+} (z_+ + \bar{z}_+) F^{-\frac{\nu}{2} \sinh \theta_- \cosh \theta_-} (z_- + \bar{z}_-)} \right] \cdot \left[\frac{F^{-\frac{\nu}{2} \sinh \theta_+ \cosh \theta_+} (\bar{z}_+ + z'_+) F^{-\frac{\nu}{2} \sinh \theta_- \cosh \theta_-} (\bar{z}_- + z'_-)}{F^{-\frac{\nu}{2} \sinh \theta_+ \cosh \theta_+} (\bar{z}_+ + z_+) F^{-\frac{\nu}{2} \sinh \theta_- \cosh \theta_-} (\bar{z}_- + z_-)} \right] \cdot \left[\frac{\tilde{F}^{-\frac{\nu}{2} \sinh \theta_+ \cosh \theta_+} (z_+ + \bar{z}'_+) \tilde{F}^{-\frac{\nu}{2} \sinh \theta_+ \cosh \theta_+} (\bar{z}_+ + z'_+)}{\tilde{F}^{-\frac{\nu}{2} \sinh \theta_- \cosh \theta_-} (z_- + \bar{z}'_-) \tilde{F}^{-\frac{\nu}{2} \sinh \theta_- \cosh \theta_-} (\bar{z}_- + z'_-)} \right] \cdot \left[\frac{\tilde{F}^{-\frac{\nu}{2} \sinh \theta_- \cosh \theta_-} (z_- + \bar{z}_-) \tilde{F}^{-\frac{\nu}{2} \sinh \theta_- \cosh \theta_-} (z'_- + \bar{z}'_-)}{\tilde{F}^{-\frac{\nu}{2} \sinh \theta_+ \cosh \theta_+} (z_+ + \bar{z}_+) \tilde{F}^{-\frac{\nu}{2} \sinh \theta_+ \cosh \theta_+} (z'_+ + \bar{z}'_+)} \right] ,$$

where $z_- = x - v_- t$, $z_+ = x - v_+ t$. In these expressions F and \tilde{F} are again ratios of Euler's elliptic ϑ -functions. The model system is invariant under reflection at either $x = 0$ (the horizontal axis) or at $x = L/2$ (the vertical axis). The scaling dimensions of a tunneling process depends in general on the edge points that are linked. For example one can see that the quasiparticle-quasiparticle correlation function and the scaling dimensions of the tunneling process remain unchanged under the shift $z, z' \rightarrow z + L/2, z' + L/2$, but the roles of g_1 and g_2 are interchanged by the shift $z, z' \rightarrow z + L/4, z' + L/4$. We have concentrated here on tunneling across a horizontal Hall bar with a constriction defined by a vertical gate with an opening at its center. This general expression suggests that quasiparticle tunneling properties can

be altered in interesting ways by changing the sample geometry. One example is that the static correlation function scaling dimension for $0 \simeq x' \ll x \ll L$ and $g_1 = g_2$, $d = \nu(4 + K + 3/K)/8$ where $K = \exp(-2\theta_+)$, compared to the standard LL expression $d = (\nu/2)(K + 1/K)$. for the standard LL.

The edges of fractional incompressible quantum Hall states are unique in that they maintain non-Fermi liquid behavior independent of the details of interactions along the edge. Their non-Fermi liquid behavior arises most fundamentally from the appearance of the fractional filling factor ν in the density-fluctuation commutators. For this reason it is usually expected that edge state properties, for example the scaling dimension of quasiparticle tunneling, should be universal. Experiments nevertheless find that quasiparticle tunneling is non-universal and often irrelevant, as noted by Rosenow and Halperin¹². In this paper we have proposed that this behavior arises from the character of interactions near quantum point contacts created by narrow split gates. We have shown that interactions between edge waves approaching and leaving the quantum point contact, tend to suppress quasiparticle tunneling. Our numerical estimates, based on the geometry of a sample studied in a recent paper by Roddaro et al.¹¹, suggest that this explanation is plausible. Since the effect requires that the split gate be narrow over a long distance, it should be possible to test this explanation experimentally. We emphasize that our quasiparticle-tunneling theory applies only in the case of an open point contact, in which the $\nu = 1/3$ incompressible fractional Hall regime is continuous from one side of the point contact to the other, and our electron-tunneling theory only to strongly pinched contacts. We expect that non-universal interaction effects will also play a role in the interesting intermediate regime¹⁶ in which incompressible regions with smaller filler factors may be formed inside the point contact. Finally, we remark that non-universal interaction effects will also play a role in samples with a simple long-thin Hall bar geometries, if the temperature can be reduced well below the energy of an edge wave with a wavelength equal to the *width* of the Hall bar. For example, for Hall bars with a width of $\sim 1\mu\text{m}$, non-universal effects due to g_2 interactions across the width of the Hall bar should start to become important below $\sim 10K$ on a $\nu = 1$ plateau.

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